# Rules of indices

**A LEVEL LINKS**
**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

## Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the $n$th root of $a$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

## Examples

**Example 1** Evaluate $10^0$

| | |
|---|---|
| $10^0 = 1$ | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate $9^{\frac{1}{2}}$

| | |
|---|---|
| $9^{\frac{1}{2}} = \sqrt{9}$<br>$= 3$ | Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ |

**Example 3** Evaluate $27^{\frac{2}{3}}$

| | |
|---|---|
| $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$<br>$= 3^2$<br>$= 9$ | **1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$<br>**2** Use $\sqrt[3]{27} = 3$ |

**Example 4** Evaluate $4^{-2}$

| | |
|---|---|
| $4^{-2} = \frac{1}{4^2}$<br>$= \frac{1}{16}$ | **1** Use the rule $a^{-m} = \frac{1}{a^m}$<br>**2** Use $4^2 = 16$ |

**Example 5** Simplify $\frac{6x^5}{2x^2}$

| | |
|---|---|
| $\frac{6x^5}{2x^2} = 3x^3$ | $6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$ |

**Example 6** Simplify $\frac{x^3 \times x^5}{x^4}$

| | |
|---|---|
| $\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$<br>$= x^{8-4} = x^4$ | **1** Use the rule $a^m \times a^n = a^{m+n}$<br>**2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$ |

**Example 7** Write $\frac{1}{3x}$ as a single power of $x$

| | |
|---|---|
| $\frac{1}{3x} = \frac{1}{3}x^{-1}$ | Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged |

**Example 8** Write $\frac{4}{\sqrt{x}}$ as a single power of $x$

| | |
|---|---|
| $\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$<br>$= 4x^{-\frac{1}{2}}$ | **1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$<br>**2** Use the rule $\frac{1}{a^m} = a^{-m}$ |

edexcel

# Practice

**1** Evaluate.

**a** $14^0$ **b** $3^0$ **c** $5^0$ **d** $x^0$

**2** Evaluate.

**a** $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

**3** Evaluate.

**a** $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

**4** Evaluate.

**a** $5^{-2}$ **b** $4^{-3}$ **c** $2^{-5}$ **d** $6^{-2}$

**5** Simplify.

**a** $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

**c** $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3y^2}{14x^5y}$

**e** $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

**g** $\dfrac{\left(2x^2\right)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

**6** Evaluate.

**a** $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

**d** $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

**7** Write the following as a single power of $x$.

**a** $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

**d** $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

Pearson

**8** Write the following without negative or fractional powers.

**a** $x^{-3}$ **b** $x^{0}$ **c** $x^{\frac{1}{5}}$

**d** $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

**9** Write the following in the form $ax^n$.

**a** $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

**d** $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

## Extend

**10** Write as sums of powers of $x$.

**a** $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$