Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2-4x-21}{2x^2+9x+9}$

$\frac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$ So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) **3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$ So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) **7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\frac{x^2-4x-21}{2x^2+9x+9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \frac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

Hint

Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2 + 4x}{x^2 - x}$ **b** $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$

c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$ **d** $\dfrac{x^2 - 5x}{x^2 - 25}$

e $\dfrac{x^2 - x - 12}{x^2 - 4x}$ **f** $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$ **b** $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$ **d** $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$